FALL 2021 - MATH 260 (18164) - TEST 1

Problem 1

Given $f(x) = \begin{cases} x^2 - 3x & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } 0 < x \leq 15. \\ -1 & \text{if } x > 15. \end{cases}$

evaluate

(a) $f(-8)$

$$f(-8) = (-8)^2 - 3(-8)$$

$$= 64 + 24$$

$$= \underline{\underline{88}}$$

b) $f(9)$

$$f(9) = \sqrt{9}$$

$$= \pm 3 \text{ ie } 3 \text{ or } -3.$$

(c) $f(100)$

$$= \underline{\underline{-1}}$$

Problem 2.

(a) Local maximum occurs at:

$(-2, 6)$, $(2, 10)$

(b) Local minimum:

$(-8, -4)$, $(0, 0)$, $(5, 0)$

(c) Intervals of Increasing:

$(-8, -2)$, $(0, 2)$, $(5, 7)$

Intervals of decreasing:

$(-10, -8)$, $(-2, 0)$, $(2, 5)$

Problem 3.

Domain of:

a) $g(x) = \dfrac{x+2}{x^2 - 2x - 35}$.

Soln.

$x^2 - 2x - 35 \neq 0$.

$P = -35$	$x^2 + 5x - 7x - 35 \neq 0$.	$x - 7 \neq 0$.
$S = -2$.	$x(x+5) - 7(x+5) \neq 0$.	$\Rightarrow x \neq 7 \ldots\ldots (i)$
$N.O = 5, -7$	$(x-7)(x+5) \neq 0$.	$x + 5 \neq 0$
		$\Rightarrow x \neq -5 \ldots (ii)$

Domain $= \{ x \mid x \neq -5, -7 \}$ OR $(-\infty, -5) \cup (-5, -7) \cup (-7, \infty)$

b) $f(x) = \dfrac{x^2}{\sqrt{8-2x}}$

Soln.

$8 - 2x > 0$

$\therefore \dfrac{8}{2} > \dfrac{2x}{2}$

$4 > x$ or $x < 4$

$\therefore D_f = \{ x \mid x < 4 \}$

OR

$(-\infty, 4)$

Problem 4

Different quotient of: $f(x) = x^2 + 3x - 1$

$f(a+h) = (a+h)^2 + 3(a+h) - 1$

$= a^2 + 2ah + h^2 + 3a + 3h - 1$

$f(a) = a^2 + 3a - 1$

$\therefore \dfrac{f(a+h) - f(a)}{h} = \dfrac{(a^2 + 2ah + h^2 + 3a + 3h - 1) - (a^2 + 3a - 1)}{h}$

$$= \frac{2ah + h^2 + 3h}{h}$$

$$= \frac{\cancel{h}(2a + h + 3)}{\cancel{h}}$$

$$= 2a + h + 3.$$

Problem 5.

Inverse of

$$f(x) = \frac{x^5 - 3}{2}$$

soln.

$$y = \frac{x^5 - 3}{2}.$$

$$x = \frac{y^5 - 3}{2}.$$

$$2x + 3 = y^5.$$

$$y = \sqrt[5]{2x + 3}.$$

$$\therefore f^{-1}(x) = \sqrt[5]{2x + 3}.$$

Verifying

$$f(f^{-1}(x)) = \frac{\left(\sqrt[5]{2x+3}\right)^5 - 3}{2} = \frac{(2x+3) - 3}{2} = \frac{2x}{2} = x.$$

$$\therefore f(f^{-1}(x)) = x.$$

Again

$$f^{-1}(f(x)) = \sqrt[5]{2\left(\frac{x^5-3}{2}\right)+3} = \sqrt[5]{x^5-3+3} = \sqrt[5]{x^5} = x$$

$$\therefore f^{-1}(f(x)) = x.$$

$\square$

Problem 6.

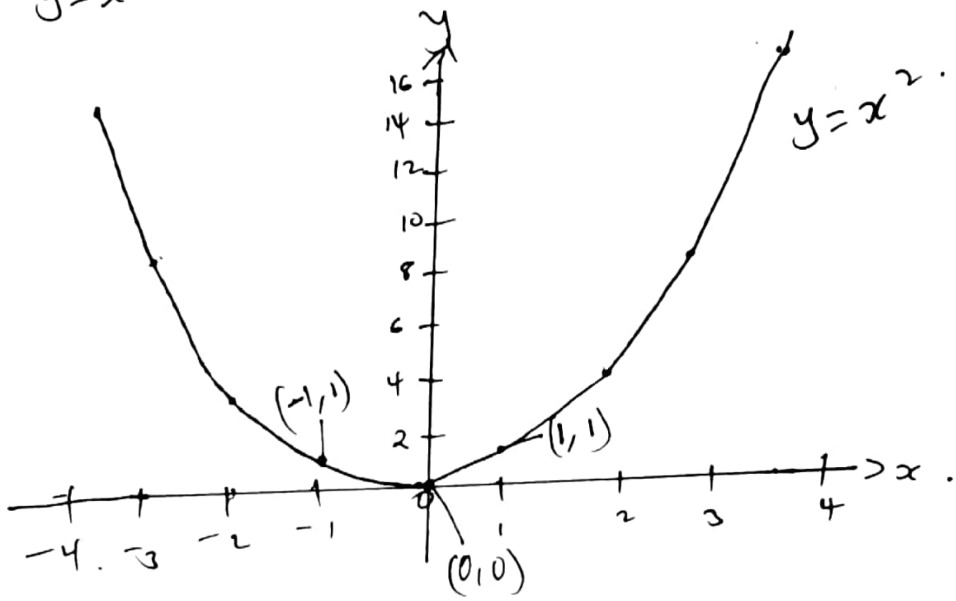
Sketch the graph of ~~z~~ $f(x) = -2(x-1)^2+3$

Starting with $f(x) = x^2$.

Solution

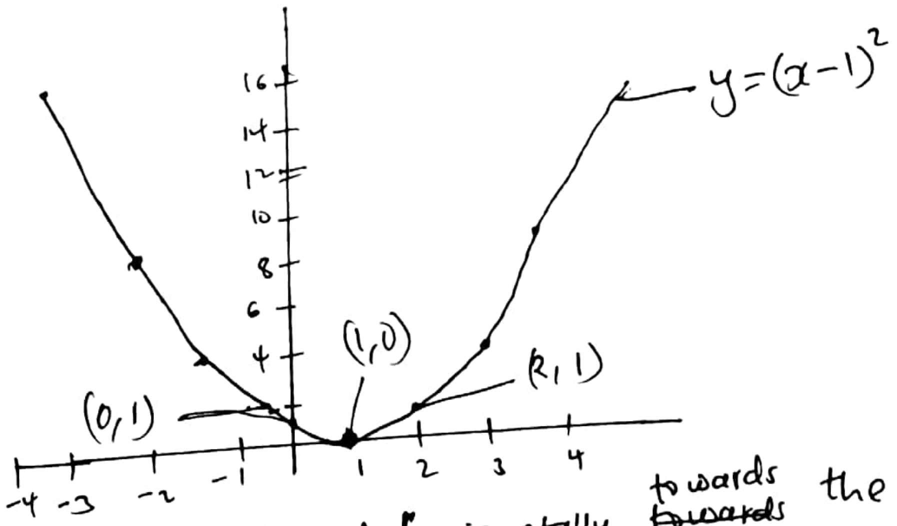
Step 1 Sketch of $y = x^2$.

x	x^2
1	1
2	4
3	9
4	16.
-1	1
-2	4
-3	9
-4	16.



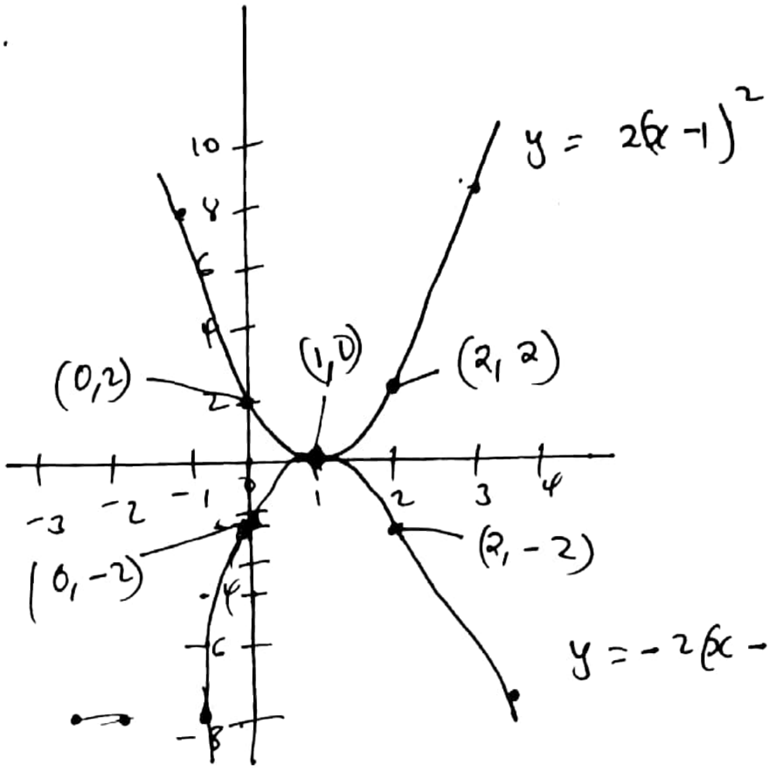
Step 2.

x	$x-1$	$(x-1)^2$.
0	-1	1
1	0	0
2	1	1
3	2	4
-1	-2	4
-2	-3	9
-3	-4	16.
4	3	9



The graph is shifted horizontally towards the right direction by 1 unit.

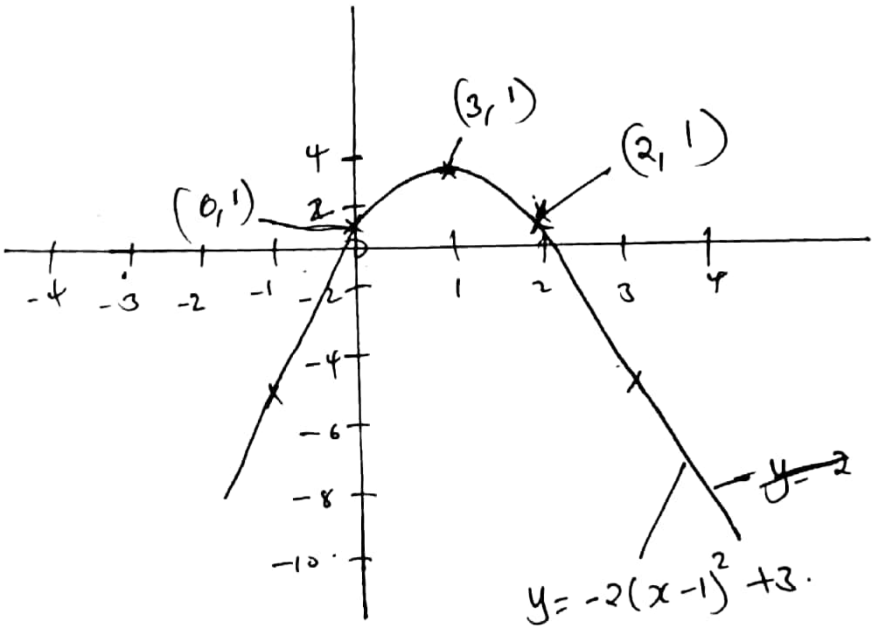
x	$-2(x-1)^2$	$2(x-1)^2$
0	-2	2
1	0	0
2	-2	2
3	-8	8
-1	-8	8
-2	-18	18
-3	-32	32
4	-18	18



The graph is stretched vertically on a scale factor of 2 then reflected along the y axis.

<u>Finally</u>

x	$-2(x-1)^2 + 3$
0	1
1	3
2	1
3	-5
-1	-5
-2	-5



The graph is finally shifted upwards by 3 units.

Problem 7

sketch $f(x) = x^2(x+2)(x-2)$

Solution:

$(x+2)(x-2) = x^2 - 4$

$x^2(x^2-4) = x^4 - 4x^2.$

$f(x) = x^4 - 4x^2.$

Step 1

End behaviour and number of turning points.

N.o of turning points $= 4 - 1 = 3.$

Behaviour : $y \to \infty$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$

Step 2: Intercepts.

at x- intercept, $y = 0.$

$x^4 - 4x^2 = 0 \iff x^2(x^2-4) = 0.$

The zeros are: $x = 0$ of multiplicity 2 $\Rightarrow (0,0)$

$x = 2$ of multiplicity 1 $\Rightarrow (2, 0)$

$x = -2$ of multiplicity 1 $\Rightarrow (-2, 0)$

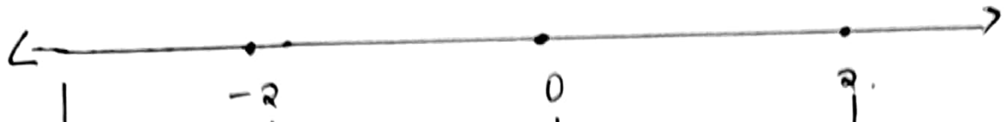
at y-intercept, $x = 0.$

$y = (0)^4 - 4(0)^2$

$= 0.$

$\Rightarrow (0,0)$

Step 3



x	-3	-1	1	3
$f(x)$	45	-3	-3	45
Point	$(-3, 45)$	$(-1, -3)$	$(1, -3)$	$(3, 45)$
Location	Above	Below	Below	Above